Closing Today: HW_5A,5B (7.1,7.2)
Closing Fri: HW_5C (7.3)

Entry Task: Evaluate

$$\int \sqrt{9 + x^2}\,dx$$

7.3 Trig. Substitution

Steps-by-step:

1. Substitute as directed below.
   Simplify (should eliminate root)
   Don't forget dx.
2. Use 7.2 methods for trig integrals.
3. Draw a triangle and return to x.

CASE	**SUBSTITUTION**
$a^2 - x^2$	$x = a\sin(\theta)$
$a^2 + x^2$	$x = a\tan(\theta)$
$x^2 - a^2$	$x = a\sec(\theta)$

Example:

$$\int \frac{\sqrt{x^2 - 16}}{x}\,dx$$

Important Application

Area under a circle

$$\int \sqrt{4 - x^2}\, dx$$

Perfect Squares

$(x+3)^2 =$

$(x-5)^2 + 2 =$

Example:

$$\int \frac{1}{\sqrt{4x^2 - 32x + 100}}\, dx$$

Completing the Square

$x^2 + 16x$ =

$3x^2 - 12x$ =

Given $\sqrt{ax^2 + \boldsymbol{bx} + c}$
complete square!

- *Factor out "a"*
- *Add/subtract half-middle squared*

Example:

$$\int \frac{x}{\sqrt{16 - 6x - x^2}}\, dx$$

7.4 Preview (Partial Fractions)

Goal: General method for rational functions.

$$\int \frac{x^3 + 4x - 4}{x^2(x^2+4)}\,dx = \int \frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^2+4}\,dx$$

Motivation:

$$\int \frac{x^3 + 4x - 4}{x^2(x^2+4)}\,dx \quad = ??$$

We will learn to break up the fraction into:

$$\frac{x^3 + 4x - 4}{x^2(x^2+4)} = \frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^2+4}$$

Then we integrate each part separately.

Partial Fraction Decomposition

Step 0: Is the fraction *reduced*?

We mean the highest power on top is strictly smaller than the highest power on bottom.

If yes, move to step 1.

If not, divide, then move to step 1.

Example:

$$\int \frac{x^2 + x}{x + 3} dx$$

A fraction skill:

Reduce the "improper" fraction into a simplified mixed fraction:

$$\frac{576}{11} = ? + \frac{?}{11}$$

Partial Fractions Method Summary

Step 0: Reduce (if needed), see last page.

Step 1: Factor Denominator.
Write out decomposition below:

i) Distinct Linear:

$$\frac{x^2-3}{x(x-1)(x+4)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+4}$$

ii) Repeated Linear:

$$\frac{5+2x}{(x+3)(x-2)^3} = \frac{A}{x+3} + \frac{B}{x-2} + \frac{C}{(x-2)^2} + \frac{D}{(x-2)^3}$$

iii) Irreducible Quadratic:

$$\frac{4x}{(x+1)(x^2+9)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+9}$$

Step 2: Solve for A, B, C ….

Step 3: Integrate

All the integrals in this section look like these:

$$\int \frac{1}{2x+5}\,dx = \frac{1}{2}\ln|2x+5| + C$$

$$\int \frac{1}{(x-4)^2}\,dx = -\frac{1}{x-4} + C$$

$$\int \frac{1}{(x+7)^3}\,dx = -\frac{1}{2}\frac{1}{(x+7)^2} + C$$

$$\int \frac{1}{x^2+9}\,dx = \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$$

$$\int \frac{x}{x^2+9}\,dx = \frac{1}{2}\ln|x^2+9| + C$$

The method uses algebra to rewrite ***any*** rational function as a sum of the integrals like those above.